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Stochastic Predictive Freeway Ramp Metering from Signal Temporal Logic Specifications

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Abstract— **We propose a ramp metering strategy capable of treating exogenous arrivals as random variables since freeway network arrivals are stochastic by nature. In order to express desired temporal properties of the network, we adopt Signal Temporal Logic (STL) as our specification language and present a general framework for synthesizing controllers for piecewise affine systems subject to stochastic uncertainties. We synthesize controllers that satisfy these stochastic STL specifications through sample average approximation techniques. We further showcase our approach for a freeway ramp metering example, we use sampling techniques to obtain ramp flows that minimize the expectation of the total travel time.**

I. INTRODUCTION

In recent years, increase in traffic congestion and delay has been recognized as an undesirable phenomenon, experienced by almost every commuter in the urban and metropolitan areas. This increase in traffic congestion necessitates the development of traffic management and control tools alleviating congestion, leading to a reduction in the costs incurred on a traffic network such as delays, greenhouse gas emissions, and fuel consumption. This alleviation can be achieved either by enhancing the current infrastructure, or devising management and control algorithms capable of increasing network throughput with the existing infrastructure. As expansion of infrastructures is a prolonged and high–priced process, the latter plays a key role in ameliorating traffic conditions.

The focus of this work is on freeway traffic networks, while the approach can be generalized to other transportation networks including urban arterials. Among possible operational strategies in freeway traffic control, ramp metering is proven to be effective and crucial [1]. Because of the effectiveness of ramp metering in reducing freeway traffic congestion, a large amount of research has been conducted on development of ramp metering algorithms ranging from classical control tools to synthesis from formal specifications. A recent approach toward traffic control is employment of formal methods techniques [2], [3], which allows the control algorithm to consider a richer class of objectives such as safety, reachability and liveness. Nonetheless, a key assumption common in the ramp metering algorithms proposed in the literature is that vehicular arrivals are deterministic and known a priori, which might not be the case in real time scenarios and applications. Vehicular arrivals are random by nature; in other words, underlying distribution of arrivals extracted from the available historical data might be the only available information about exogenous arrivals.

In this work, we aim to synthesize ramp metering controllers from temporal logic specifications while treating the vehicular arrivals as stochastic signals with known distributions. Notice that some characteristics of traffic networks, important to individual drivers, can only be captured by taking the randomness of vehicular arrivals into account. For instance, minimizing variations of Total Travel Time of the network can be an attractive and advantageous criteria as it reduces the uncertainties experienced by the drivers [4]. Particularly, our work fills the gap between inherent random nature of exogenous traffic arrivals and inadequacies of existing temporal–logic–based ramp metering algorithms to capture the uncertainties arising from this randomness.

We use *Signal Temporal Logic* (STL) [5] in a probabilistic framework, allowing for encoding rich and complex specifications of random continuous variables. Due to the nonlinearities of freeway traffic dynamics, we introduce a sampling–based technique for synthesizing controllers from STL specifications. Our framework is not specific to traffic networks, it can be viewed as a generalization of [6], where STL properties over deterministic and random variables can be handled even for piecewise affine systems, and we go beyond the restriction to the very specific form of chance constraints utilized in [6].

Our contributions in this paper are a threefold:

- Incorporating any stochastic signal in the framework of STL.
- Synthesizing controllers that optimize for the expected cost function, while the trajectories satisfy the stochastic and deterministic STL properties.
- Providing a freeway traffic network example, where we synthesize metering rates in the presence of stochastic arrivals.

The organization of this paper is as follows. In section II we provide an overview of the literature on ramp metering techniques. We then define the dynamical systems framework in section III. Section IV explains the fundamentals of freeway systems. Section V is a review of STL framework. Sections VI and VII present formulation of the problem and control synthesis procedure. Finally, in sections VIII and, IX we conclude our work by applying our method to an

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example of freeway traffic networks illustrating our approach and description of future directions.

II. PRIOR WORK

The simplest possible ramp metering scheme is fixed–time control [1], where vehicles are allowed to enter the freeway in a fixed proportion of time intervals. In [7], a simple local feedback controller for ramp metering is proposed where local density is regulated around a set–point. Another class of popular ramp controllers is the model predictive control laws, seeking for the optimal ramp flows in a sense meaningful to freeway traffic networks [8], [9], [10]. In these works, the control either optimizes for a network performance measure or forces the system to achieve a desired equilibrium point.

A modern view in synthesizing controllers for dynamical systems including transportation networks, is to exploit powerful tools of temporal logic in encoding desired properties of systems [11], [2], [12]. In all these works, the assumption is that the system is subject to deterministic bounded uncertainties, and the control is synthesized for the worst case uncertainty. However, in transportation networks such an approach might frequently lead to infeasibility of control as there exist arrival profiles for which the control cannot satisfy the properties of interest, pointing us to the need for finding control laws in a stochastic setting.

Among possible synthesis techniques, model predictive control with temporal logic specifications has been successfully implemented [13], model predictive control with signal temporal logic is shown to have a promising performance for cyber–physical systems containing continuous states using continuous-time, real-valued signals [14]. Recently, probabilistic variants of signal temporal logic has been proposed [11], [15]. In [11], a cost of interest is optimized subject to chance constraints; whereas, the probability of satisfying a STL property is maximized in [15]. Model predictive control for systems with random uncertainties requires solving stochastic optimization problems. Under certain assumptions on the distributions, constraints and cost functions, analytical solutions can be derived for some specific classes of optimization problems [16]. However, in general, stochastic optimization problems are hard to solve. In order to be able to deal with the general class of stochastic nonlinear optimizations, sampling–based techniques are proven to be appropriate alternatives [17], [18].

III. HYBRID DYNAMICAL SYSTEMS

Consider a discrete time deterministic dynamical system of the form:

$$x_{t+1} = f(x_t, u_t, w_t), \tag{1}$$

where $x_t \in \mathcal{X}$ is a signal containing the continuous and discrete states of the system, $u_t \in U$ is the signal of continuous and possibly discrete control inputs, and $w_t \in \mathcal{W}$ contains the continuous uncertainties affecting system dynamics. $\mathcal{X} \subseteq \mathbb{R}^{n_c} \times Q^S$, $U \subseteq \mathbb{R}^{m_c} \times Q^I$ and $\mathcal{W} \subseteq \mathbb{R}^{r_c}$, where n_c is the number of continuous states, m_c is the number of continuous control inputs, Q^S and Q^I are the set of discrete states and inputs, and r_c is the number of continuous uncertainties respectively. Assuming that the initial condition of the system is x_0, a finite horizon run of the system is defined as $\xi^H(x_0, \mathbf{u}^H) = (x_0, u_0), (x_1, u_1), \cdots, (x_{H-1}, u_{H-1})$ for horizon H. Here $\mathbf{u}^H$ is a finite sequence of strategies $u_0, \ldots, u_{H-1}$ leading to the sequence (trajectory) ξ^H. The notation for stochastic counterparts of these quantities will be introduced later.

IV. TRAFFIC NETWORK DYNAMICS

In regards to designing traffic controllers, macroscopic models are widely used since they deal with large-scale properties of vehicular networks rather than intricacies of lower impact on the wide-ranging performance of a network. In this work, we use Asymmetric Cell Transmission Model (ACTM) [9] which is known to be an appropriate representative of freeway traffic as it can capture merging of onramp flows properly.

In this section, we present a description of ACTM. While using ACTM, the assumption is that freeway is divided into several segments such that each segment has at most one onramp and one offramp as depicted in Figure 1. For the sake of simplicity in model description, for now, assume that the arrivals are deterministic. Suppose we have N segments, for each segment j, segment states are defined as:

- n_j : Number of vehicles stored in segment j, (density of vehicles).
- l_j : Number of vehicles queuing on the onramp corresponding to the segment j.

The onramp flows decided by the controller are the vehicular flows entering the freeway mainlines through its onramps:

- r_j : Number of vehicles entering segment j through its onramp.

In addition to onramp flows, we need to define mainline flow at each segment:

- f_j : Number of vehicles leaving segment j, moving towards the downstream segment $j+1$.

Note that since some of the segments might have offramps, we need to define the exiting flow of segments:

- s_j : Flow of vehicles leaving segment j through its offramp.

Moreover, for each onramp, the exogenous entering flow is denoted by d_j and defined as:

- d_j : Exogenous vehicular arrivals to the onramp belonging to segment j.

The dynamics and update rule of segment states can be obtained simply by the mass conservation law:

$$n_j(k+1) = n_j(k) + f_{j-1}(k) + r_j(k) - f_j(k) - s_j(k), \tag{2}$$

$$l_j(k+1) = l_j(k) + d_j(k) - r_j(k). \tag{3}$$

Remark 1: For the very first upstream segment $j = 1$, the upstream mainline flow f_0 is an exogenous arrival flow.

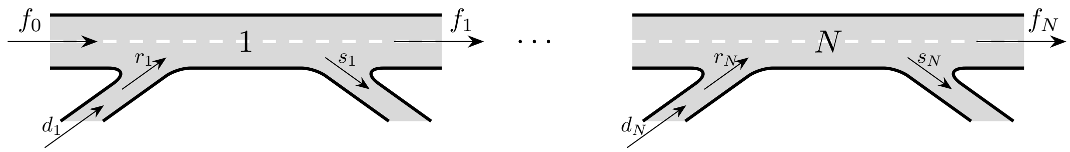

Fig. 1: Schematic of Freeway Segmentation

In order for equations (2) and (3) to represent traffic dynamics, the mapping between f_j's and n_j's is required. This mapping can be fully described provided that the following parameters are defined and calibrated for each segment:

- v_j : Normalized free–flow speed in segment j.
- w_j : Normalized congestion wave speed in segment j.
- β_j : Split ratio of segment j, the fraction of vehicles leaving segment j through its offramp: $s_j(k) = \beta_j\,(f_j(k) + s_j(k))$.
- $\bar{f}_j$: Mainline capacity of segment j, defined as the maximum number of vehicles that can leave segment j.
- $\bar{r}_j$: Onramp capacity of segment j, defined as the maximum number of vehicles that can leave the onramp of segment j.
- $\bar{n}_j$: Jam density in segment j that is the maximum number of vehicles that can be accommodated in the mainline of segment j.

Remark 2: For the segments without an offramp, one could assume $\beta_j = 0$. Also, for the segments without onramps, one could simply assume that $\forall k, r_j(k) = 0$.
A standard approach taken in transportation engineering for modeling this mapping is that the flow is related to the densities and onramp flows by:

$$f_j(k) = \min\{\bar{\beta}_j(k)v_j\,(n_j(k) + r_j(k))\,, \\ w_{j+1}(k)\,(\bar{n}_{j+1} - n_{j+1}(k) - r_{j+1}(k))\,,\, \bar{f}_j(k)\}, \tag{4}$$

where $\bar{\beta}_j(k) = 1 - \beta_j(k)$. The first term inside the minimization in equation (4) is interpreted as the number of vehicles intending to leave segment j; while, the second term accounts for available space downstream for accommodating upstream flow.

The minimization in equation (4) implies that freeway dynamics is piecewise affine, and; as a result, a nonlinear system. In other words, the dynamics of each segment can be viewed as a hybrid system of discrete modes (Free–Flow, Congestion) with continuous linear dynamics in each discrete mode. This nonlinearity leads to propagation of congestion to upstream segments once a downstream segment is congested, highlighting the important role of ramp metering in avoiding congestion creation.

In the control design procedure for a given network, we assume that the freeway is calibrated; hence, the quantities v_j, w_j, $\bar{f}_j$, $\bar{n}_j$, β_j are known and available to the controller. It is proven that ACTM never predicts negative flows or densities, making ACTM a reasonable model of traffic behavior. For further details on derivation of ACTM, refer to [9].

V. Signal Temporal Logic

To synthesize controllers for hybrid dynamical systems with continuous states and inputs, which is the case for freeway networks, we use *Signal Temporal Logic* (STL) as our formal modeling language. STL is a rich specification language capable of encoding properties of *continuous-time*, *real-valued* signals as opposed to Linear Temporal Logic (LTL), which requires discrete abstraction of the state space [19], [5]. STL specifications can be constructed recursively using the following grammar:

$$\varphi ::= \mu \mid \neg\mu \mid \varphi \wedge \psi \mid \varphi \vee \psi \mid \mathbf{G}_{[a,b]}\psi \mid \varphi\ \mathbf{U}_{[a,b]}\psi,$$

where μ is an atomic predicate of STL, φ and ψ are STL formulae that can be constructed through Boolean operators (negations $\neg$, conjunctions $\wedge$, and disjunctions $\vee$), and temporal operators such as $\mathbf{G}$ (*globally*) and $\mathbf{U}$ (*until*). One could also define the *eventually* operator by $\mathbf{F}_{[a,b]} = \neg\mathbf{G}_{[a,b]}\neg\psi$. A signal ξ satisfies a STL predicate μ at time t, $(\xi, t) \models \mu$, if and only if there exists a real-valued function $\mu(\xi(t))$ of the signal $\xi(t)$ such that $\mu(\xi(t)) > 0$. With a slight abuse of notation, μ represents both the predicate and the real-valued function $\mu(\xi(t))$. Similarly, $(\xi, t) \models \varphi$ indicates that a signal ξ satisfies the STL formula φ at time t. Satisfaction of STL formulae is specified through the following:

$$\begin{aligned}
(\xi,t) \models \mu &\iff \mu(\xi(t)) > 0 \\
(\xi,t) \models \neg\mu &\iff \neg((\xi,t) \models \mu) \\
(\xi,t) \models \varphi \wedge \psi &\iff (\xi,t) \models \varphi\ \wedge\ (\xi,t) \models \psi \\
(\xi,t) \models \varphi \vee \psi &\iff (\xi,t) \models \varphi\ \vee\ (\xi,t) \models \psi \\
(\xi,t) \models \mathbf{G}_{[a,b]}\varphi &\iff \forall t' \in [t+a, t+b], (\xi,t') \models \varphi \\
(\xi,t) \models \mathbf{F}_{[a,b]}\varphi &\iff \exists t' \in [t+a, t+b], (\xi,t') \models \varphi \\
(\xi,t) \models \varphi\mathbf{U}_{[a,b]}\psi &\iff \exists t' \in [t+a, t+b], (\xi,t') \models \psi \\
&\qquad \wedge \forall t'' \in [t,t'], (\xi,t'') \models \varphi
\end{aligned} \tag{5}$$

Note that there is a trade-off between the powerful capabilities of STL for expressing temporal properties of continuous signals and complexity of control synthesis from STL specifications. Among possible control synthesis approaches, model predictive control with STL constraints has demonstrated promising results [14], [20]. Although, providing global optimality is not guaranteed, the MPC approach is shown to perform well in practice.

VI. Problem Statement

Traditional STL allows representing predicates that are real-valued functions of signals $\mu(\xi(t))$ as described in the previous section. However, there are various situations, where the STL specification is defined over stochastic signals. Such stochasticity can arise from empirical data or uncertainty inherent as part of the system or environment. Previous work has considered such uncertainty as part of the specification through defining Probabilistic STL (PrSTL), where the predicates are probabilistic chance constraints [6]. Although PrSTL provides an effective structure for introducing stochastic Bayesian classifiers as part of the specification, it does not allow expressing general STL formulae with stochastic predicates. We propose a framework for encoding STL specifications over stochastic signals. Let $\Xi(t)$ denote a stochastic signal and $M(\Xi(t))$ be a real-valued function of the signal $\Xi(t)$. From now on, the upper case notations refer to to the stochastic counterpart of the quantities we have introduced so far. Then, we define satisfaction of a stochastic predicate M as follows:

$$(\Xi, t) \models M \iff M(\Xi(t)) \geq 0 \tag{6}$$

The rest of the operators including Boolean and temporal operators are defined similar to STL semantics in Equation (5), however over the stochastic predicate M. The function M consists of deterministic variables as well as stochastic signals.

Example 1: Let $\mathbf{X}(t) = [X_1(t)\, X_2(t)]^\top$ be the states of a two dimensional system and $u(t)$ be the control input, where $X_1(t)$ is a normally distributed random variable and $X_2(t)$ is a random variable whose randomness arises from the randomness of X_1. We let $u(t)$ be a deterministic variable. Then, we can define stochastic STL predicates over these stochastic and deterministic signals e.g. $M(\xi(t)) = X_1(t) + X_2(t) + u(t) \geq 0$. Additionally we might be interested in applying temporal operators over such specifications. For instance, we can seek for satisfaction of $\Phi = \mathbf{G}_{[0,5]}(X_1(t) + X_2(t) + u(t) \geq 0)$.

Note that if we only define deterministic variables, the stochastic STL predicate will be equivalent to classic STL specification, i.e, $M(\Xi(t)) = \mu(\xi(t))$.

Our goal is to synthesize controllers for piecewise affine dynamical systems subject to stochastic uncertainties with complex stochastic STL specifications. Note that once stochastic uncertainties propagate through piecewise affine dynamics, the system states will also be stochastic quantities whose distributions might be complicated; and, in general, there might not exist a closed form distribution for the randomness of system states. We propose a receding horizon control optimizing the expected value of a cost function J, while satisfying STL properties of stochastic signals with a desired probability of $1-\epsilon$. In our receding horizon control design, we still consider control inputs to be deterministic implying that we do not allow random inputs. Formally, given a dynamical system defined in equation (1), with the initial condition x_0, and a stochastic STL specification Φ, we would like to find a finite horizon strategy $\mathbf{u}^{H^*}$:

$$\begin{aligned} \mathbf{u}^{H^*} = \underset{\mathbf{u}^H}{\operatorname{argmin}} \quad & \mathbb{E}\left[J(\Xi^H(x_0, \mathbf{u}^H, \mathbf{W}^H))\right] \\ \text{subject to} \quad & \Pr\left(\Xi^H(x_0, \mathbf{u}^H, \mathbf{W}^H) \models \Phi\right) \geq 1-\epsilon, \end{aligned} \tag{7}$$

where $\mathbf{u}^H$ and $\mathbf{W}^H$ denote vectors of deterministic control inputs and stochastic uncertainties of a run for a horizon H respectively. Note that $\mathbb{E}\left[J(\xi^H(x_0, \mathbf{u}^H, \mathbf{W}^H))\right]$ is the expected value of the cost function of interest, Φ is a specification over the deterministic and stochastic signals, and consisting of the dynamics of the system Φ_{dyn} as well as the desired properties of the system Φ_{prop}. The dynamics Φ_{dyn} is to verify that runs of the system obey their underlying system evolution. (Note that specifications containing possibly stochastic signals are denoted by Φ rather than φ) As previously mentioned, in this work, we assume the control inputs $\mathbf{u}^H$ are deterministic quantities.

VII. CONTROL SYNTHESIS

In the receding horizon framework proposed in equation (7), $\Xi^H(x_0, \mathbf{u}^H, \mathbf{W}^H)$ and Φ are defined over deterministic and stochastic signals. Thus, the constraint $\Xi^H(x_0, \mathbf{u}^H, \mathbf{W}^H) \models \Phi$ cannot be encoded using the previously proposed techniques. As mentioned earlier, due to nonlinear dynamics of the system, even if the uncertainties have simple distributions like normal distributions, their propagation through the dynamics of the system will lead to random states with complicated distributions. Due to this complexity, computing the expected value and chance constraints in Equation VII is non–trivial. As an alternative, we propose to solve the optimization problem in (VII) approximately. Suppose the underlying distributions of uncertainties $\mathbf{W}^H$ are available [21]. We can use sample average approximation technique [18], where we generate $S(\epsilon)$ samples of the vector of random uncertainties, $\mathbf{W}_i^H, i = 1, \ldots, S(\epsilon)$. Then, for each realization of $\mathbf{W}_i^H$, we define:

$$\Phi_i = \Phi(x_0, \mathbf{u}^H, \mathbf{W}_i^H) \quad i = 1, \ldots, S(\epsilon). \tag{8}$$

The rest of the variables in Φ_i are defined as either stochastic signals being affected by each realization of $\mathbf{W}_i^H$ or deterministic variables that stay the same throughout different realizations of the random variables. Assuming that all samples are equally weighted, we approximate the optimization problem in equation (7) using this sampling based approach:

$$\begin{aligned} \underset{\mathbf{u}^H}{\text{minimize}} \quad & \frac{1}{S(\epsilon)} \sum_{i=1}^{S(\epsilon)} J(\Xi_i^H(x_0, \mathbf{u}^H, \mathbf{W}_i^H)) \\ \text{subject to} \quad & \Xi_i^H(x_0, \mathbf{u}^H, \mathbf{W}_i^H) \models \Phi_i \quad i = 1, \cdots, S(\epsilon), \end{aligned} \tag{9}$$

where $\Xi_i^H(x_0, \mathbf{u}^H, \mathbf{W}_i^H)$ denotes the (deterministic) trajectory resulting from the i_{th} realization of random uncertainties $\mathbf{W}_i^H$. One might argue that requiring the control $\mathbf{u}^H$ to satisfy Φ_i for all i's might be too restrictive or conservative. Nonetheless, the key point is that the smaller ϵ is (the higher the probability of specification satisfaction is), the larger the number of samples $S(\epsilon)$ is. Since we are dealing with piecewise affine systems, and as for the class of problems encountered in freeway ramp metering scenarios, we seek for optimizing the expected value of linear functions of the decision variables, we can use the bounds introduced in [18] to decide on the minimum number of required samples.

The practicality of sample average approximation arises from the fact that in Equation (9), each realization of Φ_i has an exact sampled value of the random variable $\mathbf{W}_i^H$, deterministic variables such as $\mathbf{u}^H$ and other stochastic signals affected by $\mathbf{W}_i^H$. Note that once $\mathbf{W}_i^H$ is available, other stochastic signals affected by $\mathbf{W}_i^H$ will be deterministic too. Thus, each Φ_i will be defined over only deterministic quantities and can recursively be encoded as mixed integer linear constraints as previously discussed in [14].

Example 2: Considering the STL formula in example 1, $\Phi = \mathbf{G}_{[0,5]}(X_1(t) + X_2(t) + u(t) > 0)$, we take S samples of the random variable $X_1(t)$. Then, for each sample $X_{1i}(t)$, there is a signal $X_{2i}(t)$ that is affected by the random variable $X_{1i}(t)$. We let $u(t)$ to be deterministic. Then, the optimization problem in (9) will be:

$$\begin{aligned} \underset{\mathbf{u}^H}{\text{minimize}} \quad & \frac{1}{S}\sum_{i=1}^{S} J(\xi^H(x_0, \mathbf{u}^H)) \\ \text{subject to} \quad & \mathbf{G}_{[0,5]}(X_{1i}(t) + X_{2i}(t) + u(t) > 0) \\ & \forall i = 1, \cdots, S. \end{aligned} \tag{10}$$

Here, we enforce the combination of stochastic and deterministic signals satisfy a STL specification.

Freeway Ramp Metering: In freeways, a well-admired cost function is $\sum_{j=1}^{N}\sum_{k=1}^{H}(n_j(k) + l_j(k))$ representing the total travel time of the vehicles in a horizon H. Thus, in the presence of stochastic arrivals, we wish to optimize for $\mathbb{E}\left[\left(\sum_{i=j}^{N}\sum_{k=1}^{H}(N_j(k) + L_j(k))\right)\right]$. The source of random uncertainties in freeways is the very upstream mainline arrival F_0, and $\mathbf{D} = \begin{bmatrix} D_1 & \cdots & D_N \end{bmatrix}^T$, which is the vector of exogenous arrivals of the network from the onramps. When F_0 is propagated through equation 4, vehicular densities and flows, N_j's and F_j's, become random variables with complicated distributions even with F_0 having a normal distribution which is due to the minimum operator in equation (4). This is similar to the problem of computing the statistical max of two arrival probability distributions [22] in statistic timing analysis of circuits, where techniques for approximating the moments of such distributions are proposed [23]. Using the sampling average approximation in equation (9), we solve for:

$$\begin{aligned} \underset{r^H}{\text{minimize}} \quad & \frac{1}{S(\epsilon)}\sum_{i=1}^{S(\epsilon)}\left(\sum_{j=1}^{N}\sum_{k=1}^{H}(N_{j_i}(k) + L_{j_i}(k))\right) \\ \text{subject to} \quad & \Xi_i^H(x_0, r^H, F_{0i}, \mathbf{D}_i) \models \Phi_i \quad i = 1, \cdots, M. \end{aligned} \tag{11}$$

where N_{j_i} is the density in segment j resulting from i^{th} realization of arrivals. Note that in order to ensure that the trajectories of the system $\Xi_i^H(x_0, r^H, F_{0i}, \mathbf{D}_i)$ obey its dynamics, we need to introduce dynamics constraint for each realization of samples. Since freeway dynamics is a piecewise affine one, dynamics constraint are encoded by mixed integer linear constraints. Any other desired temporal property needed to be satisfied with a desired probability can also be encoded as mixed integer linear constraints as in [14] since each Φ_i is a deterministic specification.

Remark 3: In our current synthesis procedure, we assume that at every time step of our receding horizon implementation, the current state which is the initial condition of the optimization problem of Equation (VII), x_0 is deterministically available which implies that one can measure the states of the system at every time step. If one wishes to treat initial conditions as random variables too, we need to update the probability distributions over freeway states at every time step. To this end, we can use particle filtering schemes as in [24].

Solving mixed integer linear programs is NP-hard, which incurs huge computational costs when dealing with high–dimensional systems. On the other hand, freeways normally contain many segments, making the proposed control scheme impractical when considering a stretch of freeway. However, it is shown that when there exist bottlenecks in freeways (which is in fact when ramp metering is required), freeways are divided into separated regions decoupled from each other by bottlenecks [1]. This decoupling and large time scales of freeways make our computations tractable to be implemented for each region. On the other hand, our framework can be utilized for compositional design of freeway controllers as in [25], where the issue of scalability for large networks is addressed by the notion of assume–guarantee contracts of small subnetworks. Each subnetwork synthesizes its control laws in line with the assumed temporal properties of adjacent networks. In addition, using a convex subset of STL specifications (under conjunctions and globally with convex cost functions) we do not require construcing integer variables for each constraint [6] which further reduces the complexity of the optimization problem.

VIII. Example

Consider two successive freeway segments with each segment having an onramp. We assume the upstream segment has an off–ramp as well. We wish to determine ramp flows such that the expected total travel time of the two links is minimized for a horizon of 10 time steps. We adopt $\bar{n}_1 = \bar{n}_2 = 400$, $v_1 = v_2 = 0.7$, $w_2 = \frac{1}{6}$, $\bar{f}_1 = \bar{f}_1 = 60$, $\bar{r_1} = \bar{r_1} = 30$ and $\beta_1 = \frac{1}{4}$ as our network parameters. These values are aligned with the illustrative examples in [2]. A reasonable assumption on network arrivals is that they can be modeled with Poisson Processes. This implies that at every time step, vehicular arrivals are obtained through a Poisson distribution, $P(x, \lambda)$, where the average number of vehicles entering the network at every time step is λ. Resultantly, we assume F_0, D_1 and D_2 are generated by $P(x, \lambda_{F_0})$, $P(x, \lambda_{D_1})$ and $P(x, \lambda_{D_2})$ respectively. We choose λ_{F_0} to be 40 and $\lambda_{D_1} = \lambda_{D_2}$ to be 15. We assume that $n_1(0) = n_2(0) = 50$.

An important temporal property required for freeway traffic control is that vehicular queues on the onramps get discharged infinitely often. As a result, an interesting temporal property is that ramp queues get less than a threshold infinitely often. For instnace, in this example, we desire to encode that $\Pr\left(\mathbf{GF}_{[0,5]}\, l_2 \leq 10\right) \geq 0.5$. Note that for traffic networks, enforcing the controller to satisfy a temporal

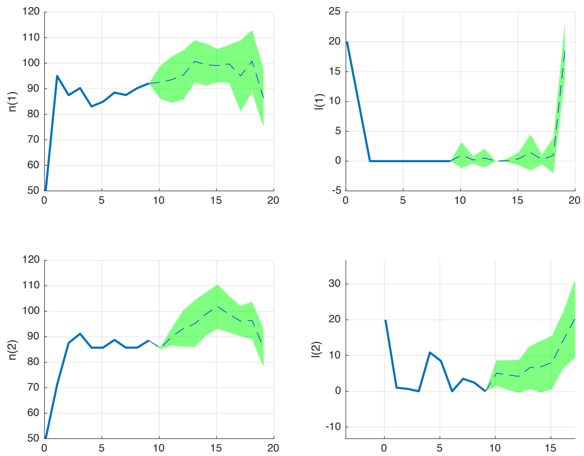

Fig. 2: Illustration of uncertainties considered by the MPC

property with high probability will lead to infeasibility issues as occurrence of inconceivable arrivals is a frequent scenario in traffic networks, highlighting the need to view transportation networks in a stochastic setting rather than deterministic systems subject to bounded uncertainties. As Figure (2) shows, the controller predicts the uncertainties which might exist for future time steps (the green area in the plot), and, based on that, it synthesizes control laws such that the queue on the second onramp is infinitely often less than 10 while picking the control such that the average Total Travel Time of the network is minimized.

IX. FUTURE WORK AND CONCLUSION

We illustrated a freeway ramp metering layout with arrivals as uncertain parameters. We showed that using sample average approximation techniques, we can encode Signal Temporal Logic specifications over stochastic signals. Like any research, our work is limited in many ways, the trade–off between the number of samples and accuracy of our algorithm is a direction to explore. We would like to define weights proportional to the likelihood of observing a sample leading to reduction in the number of samples. Similar ideas are used in particle filtering schemes.

Despite these limitations, we are excited to pursue this direction in other traffic control problems such as coordinating signalized intersections of urban arterials. In addition, our framework smooths the way for taking variance of Total Travel Time in rerouting strategies of networks. Moreover, our work can be integrated with updating the belief on distribution of arrivals as time passes so as to provide more freedom to the control.

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